Fermionic Symmetries: Extension of the two to one Relationship Between the Spectra of Even-Even and Neighbouring Odd mass Nuclei

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Abstract

In the single j shell there is a two to one relationship between the spectra of certain even-even and neighbouring odd mass nuclei e.g. the calculated energy levels of $J=0^+$ states in ^{44}Ti are at twice the energies of corresponding levels in ^{43}Ti(^{43}Sc) with $J=j=7/2$. Here an approximate extension of the relationship is made by adopting a truncated seniority scheme i.e. for ^{46}Ti and ^{45}Sc we get the relationship if we do not allow the seniority v=4 states to mix with the v=0 and v=2 states. Better than that, we get *very* close to the two to one relationship if seniority v=4 states are admixed perturbatively. In addition, it is shown that the higher isospin states do not contain seniority four admixtures.

I. INTRODUCTION

Single j-shell (j=7/2 in particular) configurations are not only simple but also offer ideal situations for realizing a wide variety of relationships in the otherwise complex spectra. Although the semi-magic Ca isotopes are quite extensively studied [1], the existence of such relations for open-shell nuclei is doubtful due to the presence of the proton-neutron interaction. However, it was noted by McCullen, Bayman and Zamick (MBZ) [2] that in a single j shell calculation (j=$f_{7/2}$) nuclei with open shells of both neutrons and protons (e.g. Scandium and Titanium Isotopes) there were in some cases striking relations in the calculated spectra of even-even nuclei and neighbouring odd-even nuclei. For example, the excitation energies of the J=0^+ states in ^{44}Ti were at twice the energies of J=j states in ^{43}Sc (or ^{43}Ti). It was further shown in MBZ technical report that the wavefunctions for the even- even and even-odd nuclei bear a striking visual relationship. The wave functions for Ti were written as

$$\psi^\alpha = \sum_{L_p L_n} D^{J_\alpha} \left(L_p L_n \right) \left[\left(j^p \right)^{L_p} \left(j^n \right)^{L_n} \right]^J \tag{1}$$

and those of Sc as

$$\psi^\beta = \sum_{L_n} C^{J_\beta} \left(L_n \right) \left[j^p \left(j^n \right)^{L_n} \right]^J \tag{2}$$

In the above equation $D^{J_\alpha} \left(L_p L_n \right)$ is the probability amplitude that in a state α with total angular momentum J the protons couple to L_p and the neutrons to L_n; a similar definition holds for $C \left(L_n \right)$.

For ^{43}Sc (J=j=7/2) and for ^{44}Ti (J=0) the relationship is

$$D^0 \left(L_p L_n \right) = C^j \left(L_n \right) \tag{3}$$

for a state in ^{44}Ti which is at twice the excitation energy of the corresponding state in ^{43}Sc.

This relationship also holds for other pairs as well e.g. $\left({}^{48}Ti, {}^{49}Ti \right)$, $\left({}^{52}Fe, {}^{53}Fe \right)$.

Note that the dimensions of the column vectors in the two cases is the same. For ^{43}Sc J=j and the possible values of L_n are 0,2,4 and 6. For ^{44}Ti J=0 and the allowed $[L_p L_n]$ states are [0,0], [2,2], [4,4] and [6,6]. In both cases the dimensions are the same i.e. four and four. This is necessary in order to have an _exact_ two to one relationship.

A comparison between theory and experiment was carried out by Zamick and Zeng [3] focussing on the excitation energies of high isospin states. For example, for ^{48}Ti the

experimental T=4 J=0 excitation energy is 17.379 MeV, while in ^{49}Ti the excitation energies of the T=7/2 J=j=7/2$^-$ state is 8.724 (the ground states have isospins T=2 and T=5/2 respectively). The deviation from a two to one relation is -0.40%. For ^{52}Fe and ^{53}Fe the excitation energies of the T=2 and T=3/2 states are respectively, 8.559 and 4.25 MeV and the percent deviation is 0.69%. Other cases are considered where there is no exact two to one ratio in the theory e.g. ^{46}Ti and ^{47}Ti where the percent deviation is -1.54% and ^{44}Ti, ^{46}Ti where the percent deviation is -0.98%. The agreement with two to one relation is surprisingly good for these pairs. However, in these nuclei the two to one ratio is not expected to hold for states of lower isospin.

In the following section we show the exactness of the two to one relationship in neighbouring pairs of nuclei with two, one particles (or holes), respectively, in j=7/2 shell. The approximate extension of the relation to certain other pairs of nuclei with more valence particles/holes is discussed in Sect.3 and the absence of seniority 4 contributions to the higher isospin states in these nuclei is shown in Sect.4. In Sect.5 higher seniority component admixtures are treated perturbatively. Finally, some additional remarks are made in Sect.6.

II. EXACT TWO TO ONE RELATION FOR [^{44}TI, ^{43}SC] (AND [^{48}TI, ^{49}TI] AND [^{52}FE, ^{53}FE])

The wavefunction of a Ti isotope can be written as

$$\psi^{J_\alpha} = \sum_{L_p L_n} D^{J_\alpha}(L_p L_n) [L_p L_n]^J \tag{4}$$

where L_p is the angular momentum of the two protons and L_n that of the neutrons. We have the normalization condition

$$\sum_{L_p L_n} \left| D^{J_\alpha}(L_p L_n) \right|^2 = 1 \tag{5}$$

The coefficients can be regarded as parts of a column vector representing the wavefunction such that $\left| D^{J_\alpha}(L_p L_n) \right|^2$ is the probability that in a given state α with angular momentum J, the protons couple to L_p and the neutrons to L_n.

To obtain the wavefunction we have to diagonalize the Hamiltonian matrix, a typical matrix element of which is

$$\left\langle \left[L'_p L'_n \right]^J |V| [L_p L_n]^J \right\rangle$$

Normally what one tries to do, and this was indeed done by MBZ [2], is to reduce this to sums over two particle matrix elements of the form $\left\langle (j^2)^J |V| (j^2)^J \right\rangle$ J=0,1,...,7. However,

we will not follow that procedure. Rather we will first consider the matrix element of the even even nucleus ^{44}Ti and we will manipulate the expression so that we can get rid off the co-ordinates of one of the particles and thus establish a relationship with ^{43}Sc (and its mirror ^{43}Ti). We will assume charge symmetry $V_{nn} = V_{pp}$.

The matrix element of the even even ^{44}Ti is written as

$$M\left(^{44}Ti\right) = \left\langle \left[\left(j^2\right)^{L_p'} L_n'\right]^J |V| \left[\left(j^2\right)^{L_p} L_n\right]^J \right\rangle$$

We can break this into a) an interaction between the protons, b) an interaction between the neutrons and c) an interaction between neutrons and protons.

For a) and b) we get

$$\left[\langle L_p |V| L_p\rangle + \langle L_n |V| L_n\rangle\right] \delta_{L_{p'} L_p} \delta_{L_{n'} L_n}$$

For the interaction between neutrons and protons we get

$$2\left\langle \left[\left(j^2\right)^{L_p'} L_n'\right]^J |V(p; neutrons)| \left[\left(j^2\right)^{L_p} L_n\right]^J \right\rangle$$

In the above we included in V only the interaction of the second proton with the neutrons, and compensate by multiplying the matrix element by a factor of two. This is justified by the fact that the wavefunction of the two protons is antisymmetric.

We can use the Racah algebra to couple the second proton to the neutrons. It is convenient to use the unitary Racah coefficients defined by

$$[[ab]j_{ab}c]^J = \sum_{J_{bc}} U\left(abJc; j_{ab}j_{bc}\right) [a[bc]j_{bc}]^J$$

They are related to the more familiar 6j symbols by

$$U\left(abcd; ef\right) = (-1)^{a+b+c+d}\sqrt{(2e+1)(2f+1)} \begin{Bmatrix} a & b & e \\ d & c & f \end{Bmatrix}$$

We get

$$V_{proton-neutron} = \{1 + (-1)^{L'-L}\} \sum_{I_x} U\left(jjJL_n'; L_p'I_X\right) U\left(jjJL_n; L_pI_X\right)$$

$$\left\langle \left[j_p \left[j_pL_n'\right]^{I_X}\right]^J |V| \left[j_p \left[j_pL_n\right]^{I_X}\right]^J \right\rangle \tag{6}$$

Since L and L′ are even for two protons in a single j shell the factor $\{1 + (-1)^{L-L'}\} = 2$.

We now specialize to J=0 states of ^{44}Ti for which L_p and L_n are equal. From the unitarity condition

$$U\left(jj0L'_n; L'_p I_X\right) = \delta_{I_X j}$$

Thus

$$V_{proton-neutron} = 2\left\langle [jL'_n]^j \,|V|\, [jL_n]^j \right\rangle$$

Invoking charge symmetry we find that the proton- proton + neutron-neutron interaction equals $2\langle L_n\,|V|\,L_n\rangle\,\delta_{L_nL_{n'}}\delta_{L_pL_n}$. But this is just twice the corresponding matrix element between two neutrons in ^{43}Sc for a state with total angular momentum J = j. We thus see that a given matrix element for the J=0 state of ^{44}Ti is twice that of the corresponding matrix element for the J=j state in ^{43}Sc. Thus the column vectors will have identical numbers and there will be a two to one ratio for the energy levels.

We show the energies and column vectors for ^{44}Ti, ^{43}Sc in Table.I, as they were originally calculated by MBZ and published in their technical report [2].

TABLE.I. Two to One Relation in ^{44}Ti,^{43}Sc.				
Eigenvalues and Wavefunctions for J=0 levels in ^{44}Ti				
Energy	0.0	6.5007	8.3449	10.8567
L_p $\quad L_n$			T=2	
0 $\quad$ 0	-0.7608	0.4006	-0.5000	0.1037
2 $\quad$ 2	-0.6090	-0.6995	0.3727	0.0317
4 $\quad$ 4	-0.2093	0.4156	0.5000	-0.7304
6 $\quad$ 6	-0.0812	0.4213	0.6009	0.6744
Eigenvalues and Wavefunctions for J=7/2 levels in ^{43}Sc				
Energy	0.0	3.2503	4.1724	5.4284
L_p $\quad L_n$			T=3/2	
7/2 $\quad$ 0	-0.7608	0.4006	-0.5000	0.1037
7/2 $\quad$ 2	-0.6090	-0.6995	0.3727	0.0317
7/2 $\quad$ 4	-0.2093	0.4156	0.5000	-0.7304
7/2 $\quad$ 6	-0.0812	0.4213	0.6009	0.6744

III. APPROXIMATE TWO TO ONE RELATIONSHIP FOR ^{46}TI AND ^{45}SC

As an example consider the pair ^{45}Sc, ^{46}Ti. The basis states for ^{45}Sc with J=j=7/2 consist of a single proton with L_p=j and four neutrons with angular momenta L_n=0,2,4,6,2*,4*,5*, where the states 2,4,6 have seniority _two_ and the states 2*,4* and 5* have seniority 4. The basis states for ^{46}Ti are [0,0], [2,2], [4,4], [6,6], and [2*,2*] and [4*,4*].

The dimension being 7 for ^{45}Sc and 6 for ^{46}Ti, it is not possible to have an exact two to one relationships.

Suppose, however, we make the approximation that for the lowest lying states we can omit the seniority _four_ admixtures. The dimensions then become 4 and 4 so there is a hope for getting a two to one relationship. In the following paragraphs we will show that this hope is realized.

The wavefunctions and energy levels for ^{46}Ti and ^{45}Sc; as calculated by MBZ [2] are shown in Table.II.

TABLE.II. Approximate Two to One Relation in ^{46}Ti,^{45}Sc.						
Eigenvalues and Wavefunctions for J=0 levels in ^{46}Ti						
Energy	0.0	5.1973	7.1207	9.2493	11.4350	12.9491
L_p L_n						T=3
0 0	0.8224	-0.3982	0.1527	-0.0724	0.1913	-0.3162
2 2	0.5420	0.5245	-0.1105	0.3756	-0.3333	0.4082
2* 2*	0.0563	0.4309	0.6819	-0.5783	-0.1082	0.0000
4 4	0.0861	-0.4461	-0.2342	-0.5244	-0.4046	0.5477
4* 4*	-0.1383	-0.4006	0.5755	0.4645	-0.5228	0.0000
6 6	-0.0127	-0.1454	0.3367	0.1686	0.6353	0.6583
Eigenvalues and Wavefunctions for J=7/2 levels in ^{45}Sc						
Energy	0.0	2.6204	3.2255	4.9559	5.5225	6.4779	6.6443
L_p L_n						T=5/2	
7/2 0	0.8210	-0.4154	0.0811	-0.0536	0.2068	-0.3162	0.0343
7/2 2	0.5434	0.5555	0.1042	0.1420	-0.4362	0.4082	0.0904
7/2 4	0.0846	-0.4740	-0.4599	-0.0533	-0.2079	0.5477	-0.4588
7/2 6	-0.0130	-0.1496	0.3570	-0.0695	0.5429	0.6583	0.3422
7/2 2*	0.0428	0.2197	0.1706	-0.9142	0.0160	0.0000	-0.2912
7/2 4*	-0.1462	-0.4540	0.6329	-0.0354	-0.6030	0.0000	0.0850
7/2 5	-0.0120	-0.1319	-0.4625	-0.3638	-0.2554	0.0000	0.7556

For J=0 states in ^{46}Ti the wavefunctions are of the form

$$\psi = \sum_{L_V} D^{0\alpha}(L, L_V)[LL_V]^0 \delta_{L,L_V}$$

i.e. the angular momentum of the two protons must equal the angular momentum of the four neutrons. As mentioned before there are two L=2 and L=4 states corresponding to seniorities v=2 and v=4.

Just as in eq.(6) in the previous section, the Hamiltonian matrix is of the form

$$\left\langle [L'L_V']^0 \,|H|\, [LL_V]^0 \right\rangle = V_{pp}^{L'}\delta_{L'L} + V\left(f_{7/2}^4\right)_\nu^{L_V}\delta_{LL'}$$
$$+2U(jj0L';l'j)U(jj0l;lj)\left\langle \left[j\,[jL']^j\right]^0 \,|V_{pn}|\, \left[j\,[jL]^j\right]^0 \right\rangle \qquad (7)$$

The last factor is equal to $< [jL']^j |V_{pn}|[jL]^j >$ i.e. the proton-neutron interaction in ^{45}Sc. The Unitary Racah coefficients are both equal to unity because of the zero on the left side of semicolon.

Consider the interaction between the neutrons. At first glance it does not seem possible that the interaction between four neutron could equal that of two protons. But there is the remarkable result discussed in De Shalit and Talmi [4] and in Talmi's more recent work [5] that for a two-body effective interaction in the $f_{7/2}$ shell if we limit ourselves to seniority v=0 and v=2 states,

$$V\left(f_{7/2}^4\right)_\nu^L = V\left(f_{7/2}^2\right)_\nu^L + constant.$$

A consequence of this result is that the seniority two states in all the nuclei described by $\left(f_{7/2}\right)^n$ configurations have nearly the same spectra. Therefore if we truncate to seniority 0 and seniority 2 states we find

$$< [LL]^0|H|[L'L']^0 > = \left(V_{pp}^L + V_{nn}^L\right)\delta_{LL'} + 2 < [jL']^j|H|[jL]^j > +constant$$

By charge symmetry $V_{nn}^L = V_{pp}^L$ and so the spectrum of ^{46}Ti will be double that of ^{45}Sc provided we limit ourselves to v=0 and v=2. This approximation should be quite good for the first few states of the two nuclei. We shall see in the next section that the situation is even better for states of higher isospin — they do not have components in which the four neutrons couple to seniority v=4.

IV. HIGHER ISOSPIN STATES

In the single j shell all but one of the J=0 states in ^{46}Ti have isospin T=1. The other state has isospin T=3. If we compare the wavefunction of this state with the higher isospin state in ^{45}Sc we see that the numbers in the column vectors are the same. Further more there are no seniority 4 neutron state components in the wavefunctions.

We can explain the result as follows. ^{46}Ti(T=3)J=0$^+$ is the double analog of the J=0$^+$ groundstate of ^{46}Ca. This nucleus has only valence neutrons. Thus the amplitudes D(L$_p$L$_n$)$^{J=0T=3}$ should be two particle fractional parentage coefficients.

$$^{46}Ca^{(J=0)} = \sum_{I_0,v,v'} \left\langle \left(j^4\right)^{I_0} v \left(j^2\right)^{I_0} v' \left| \right\} j^6 J=0v=0 \right\rangle \left[\left(j^4\right)^{I_0} v \left(j^2\right)^{I_0} v' \right]^{J=0v=0}$$

$$= \sum_{I_0,v} \left\langle \left(j^4\right)^{I_0} v \quad j \left| \right\} j^5 J=jv'=1 \right\rangle \left\langle \left(j^5\right)^j v'=1 \quad j \left| \right\} j^6 J=0v=0 \right\rangle$$

$$U\left(I_0 j(J=0)j; jI_0\right) \left[\left(j^4\right)^{I_0} \left(j^2\right)^{I_0} \right]^{J=0} \tag{8}$$

Here the one particle cfp $\left\langle \left(j^5\right)^j v'=1j \left| \right\} j^6 J=0v=0 \right\rangle$ is equal to one as the coupling of 5 particles to the sixth particle to give angular momentum zero and seniority zero state is unique and the other one particle cfp has non zero values only for seniority v=0 and v=2 only. Thus unlike in the previous section there is no necessity for truncation as only seniority v=0 and v=2 components enter, making the relation an exact one. Once again, since J=0 the U-coefficient is equal to δ_{jI_0}. Hence the 2 particle cfp $\left\langle \left(j^4\right)^{I_0} v \left(j^2\right)^{I_0} v' \left| \right\} j^6 J=0v=0 \right\rangle$ is equal to the one particle cfp $\left\langle \left(j^4\right)^{I_0} v \quad j \left| \right\} j^5 J=jv'=1 \right\rangle$. Therefore the non-zero numbers in the column vectors (or wavefunctions) for the T=3 and T=5/2 states are the same and they correspond to the non-zero values of the cfps and they can be analytically calculated [5,6] as (it should be noted that the cfps can be calculated to within an overall phase),

$$\left\langle \left(j^{n-1}\right)^0 v=0 \quad j \left| \right\} j^n J=jv'=1 \right\rangle = \sqrt{\frac{(2j+2-n)}{(n)(2j+1)}}$$

$$\left\langle \left(j^{n-1}\right)^{I_0} v=2 \quad j \left| \right\} j^n J=jv'=1 \right\rangle = -\sqrt{\frac{2(n-1)(2I_0+1)}{(n)(2j+1)(2j-1)}}$$

V. HIGHER SENIORITY ADMIXTURES IN PERTURBATION THEORY

The approximate two to one relationship for ^{46}Ti and ^{45}Sc also applies to the cross conjugate pair in which protons and neutron-holes are interchanged as well as neutrons and proton-holes. The pair in question is ^{50}Cr and ^{51}Cr. If we examine the Nuclear Data Sheets [7] we find that there is not sufficient data for the pair [^{46}Ti, ^{45}Sc] i.e. eventhough the T=3 0^+ state in ^{46}Ti is observed at 14.153 MeV, the corresponding T=5/2 $7/2^-$ state in ^{45}Sc is still missing, but there is for [^{50}Cr,^{51}Cr]. The T=3 - T=1 splitting in ^{50}Cr is 13.222 MeV and the T=5/2 - T=3/2 splitting is 6.611 MeV. This is amazing, the two to one relationship holds to four significant digits.

The closeness of the results leads us to ask if we have gone as far as one can go in the previous sections. The answer is no! From Table-II we can evaluate the calculated percent

admixtures of v=4 components in the ground states of ^{46}Ti and ^{45}Sc. The respective values are 2.232% and 2.335%. They are almost the same.

Let us therefore consider seniority 4 admixtures in perturbation theory. Suppose we have obtained approximate ground states for ^{46}Ti and ^{45}Sc by not allowing v=4 admixtures. The approximate wavefunctions will be

$$^{46}Ti \ \psi \ = \ \sum_{L;v=0,2} \tilde{D}(LL)[LL]^0$$
$$^{45}Sc \ \psi \ = \ \sum_{L;v=0,2} \tilde{D}(LL)[jL]^0$$

Let us consider the matrix element which couples seniority 4 admixtures in ^{46}Ti

$$M = \sum_{L'v=0,2} \tilde{D}(L'L') \left\langle [L'L']^0 \, |V| \, [L(Lv=4)]^0 \right\rangle$$

There will be no contribution from the proton-proton interaction because of the orthogonality of the neutron wavefunctions $< L'v \neq 4|Lv = 4 >= 0$. There will be no contribution from the neutron-neutron interaction because $< L'v \neq 4|V|Lv = 4 >= 0$ i.e. as mentioned before seniority is a good quantum number for particles of one kind in the f$_{7/2}$ shell [5].

The only contribution is from the proton-neutron interation. Using the same techniques as in previous sections we obtain.

$$M = 2 \sum_{L'v=0,2} \tilde{D}(L'L') \left\langle [jL']^j \, |V| \, [j(Lv=4)]^j \right\rangle$$

This is exactly twice the corresponding mixing matrix element for ^{45}Sc, except for the fact that for ^{45}Sc one can have L=5 v=4, but not in ^{46}Ti. If we neglect the above difference we can use a "$V^2/\Delta E$" argument. For ^{46}Ti V and ΔE are both twice what they are in ^{45}Sc. The ground state energy shift Δ for ^{46}Ti is

$$\Delta \left(^{46}Ti \right) = \frac{[2V \left(^{45}Sc \right)]^2}{[2\Delta E \left(^{45}Sc \right)]} = 2\Delta \left(^{45}Sc \right)$$

There will be no energy shifts for the states of higher isospin because they have no v=4 admixtures.

Thus in ^{45}Sc if the unperturbed energy shift is E_0, then when v=4 admixtures are added perturbatively the shift is $E_0 - \Delta$. In ^{46}Ti the shift is $2E_0 - 2\Delta$. Thus the two to one ratio is preserved.

In a complete matrix diagonalization there will be deviation from the 2 to 1 ratio because of diagonal energy shifts and because of the previously mentioned L=5 v=4 component which is present in ^{45}Sc but not in ^{46}Ti.

Nevertheless, the two to one ratio holds better than we would expect from merely truncating in seniority — it holds when higher seniority states are admixed in perturbation theory.

VI. ADDITIONAL REMARKS

As mentioned in [3], Rubby Sherr [8] noted that a simple interaction $a + bt_1 \cdot t_2$ where a and b are constants, will lead to a two to one ratio for excitations of states of higher isospin, not only in the nuclei covered thus far but also for the pairs ^{44}Ti, ^{45}Ti and ^{46}Ti and ^{47}Ti. Indeed the percent deviation for these nuclei is small -0.98% and -1.54% respectively. However, for these nuclei there is no two to one relationship predicted for the states of lower isospin and the counting of states is quite different. In ^{44}Ti there are 4 J=0 states in the $f_{7/2}$ shell whilst in ^{45}Ti there are 17 J=j states. The corresponding values for ^{46}Ti and ^{47}Ti are 6 and 17.

There is one comment worth making about the seniority content of the J=0_1^+ state in ^{46}Ti and the j=7/2$^-$ state in ^{45}Sc. While the L_n=2 v=2 probability in the states is much larger than the L_n=2 v=4, we find that for for L_n=4, the v=4 probability is somewhat larger than v=2. This can be understood in terms of boson models. Roughly speaking, the L_n=2, v=2 state corresponds to a single d boson whereas the L_n=2 v=4 state corresponds to two d bosons coupled to L_n=2. It is not surprising that one d boson admixture in the ground states should be larger than the two d boson admixture.

For L_n=4 the v=2 state corresponds to one g boson whilst the v=4 state corresponds to two d bosons [6]. The g boson is at about twice the energy of the d boson and this fact causes the admixture of two d bosons to be comparable to the amount of one g boson in the ground state.

Acknowledgements

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